SIMULATION OF FLUID FLOW INSIDE POROUS STRUCTURES USING LATTICE BOLTZMANN MODEL

L.O. E. Santos[1], P.C. Philippi[2•] M.C. Damiani[3]

(+) Porous Media and Thermophysical Properties Laboratory (LMPT)
Mechanical Engineering Department. Federal University of Santa Catarina
88040-900 Florianópolis, SC, Brazil.

(*) Engineering Simulation and Scientific Software (ESSS)
Parque Tecnológico de Florianópolis - Rodovia SC 401 km 001
88030-000 - Florianópolis - SC – Brazil

Abstract

LB methods were recently developed for the numerical integration of the Navier-Stokes equation based on discrete Boltzmann transport equation. Derived from the lattice gas method, LB is a powerful alternative to the standard methods in computational fluid dynamics. In recent years, it has received much attention and has been used in several applications like simulations of flows through porous media, turbulent flows and multiphase flows. Present paper presents the results of using Lattice-Boltzmann methods (LB) for simulating 2D von Karman vortex street, simulation of fluid flow inside complex three-dimensional porous structures and comparison between simulation and experimental values of intrinsic permeability for Berea and other petroleum reservoir sandstone

---

[1] *Luis Orlando Emerich dos Santos has a BS degree in Electrical Engineering and a MSc degree in Physics. He is presently making his Doctoral studies in Porous Media and Thermophysical Properties Laboratory (LMPT) which include the use of lattice gas automata (LGA) for the analysis of single and two-phase flow displacements in 3D porous representations.*

[2] *Paulo C. Philippi is Professor at Federal University of Santa Catarina where he heads the Porous Media and Thermophysical Properties Laboratory (LMPT). He specializes in image analysis and processing, single and multiphase flow in porous media and lattice gas automata (LGA) methods.*

• Corresponding Author (philippi@lmpt.ufsc.br)

[3] *Marcos Cabral Damiani is a software development engineer from the Engineering Simulation and Scientific Software (ESSS) staff. Presently, he is working in the software development of Imago at Porous Media and Thermophysical Properties Laboratory (LMPT).*

1. Introduction

Although computer non-expensive, considering processing operations and resident memory requirements, lattice gas cellular automata (LGA) have intrinsic flutuactions related to the use of boolean variables and is not suitable for describing hydrodynamics, excepting in the limit of incompressible, low Reynolds number flows[1]. In fact, the equilibrium distribution for the expected values of the boolean variables, used in these models, is a Fermi-Dirac[2],[3] distribution, producing non-physical effects for intermediates and large Reynolds number.

Lattice Boltzmann method (LB) was originated from the ensemble average of LGA microscopic dynamic equation[5][6]. In this aspect:

i) it is based on regular discrete lattices, maintaining propagation and collision steps,

ii) it works with real variables instead of boolean variables, which are interpreted as the probability $N_i(\mathbf{X},T)$ of finding a particle in the direction i of a given lattice site $\mathbf{X}$, at time T.

The main feature of Boltzmann method, distinguishing this method from a method based on statistical ensemble averages of boolean models, is that the collision term is written to give the expected macroscopic behavior. In this sense, LB method can be seen as a common numerical method, based on relaxation equations, written to reproduce the hydrodynamic behavior of the flow problem we want to solve.

In this way LB method is not limited to low Reynolds number flows, being suitable to the study of flow problems where inertial effects are important. In fact, it has been used in the last decade in the study of instable flows with periodic fluctuations (von Karman vortex street).

In present work, LB method is presented and applied to the study of fluid flow simulation through porous structures of complex geometry. Simulation results are presented and compared with experimental values in the prediction of intrinsic permeability of Brazilian Sandstones.

2 - Model

Microscopic Dynamics

Three-dimensional physical space is considered to be a cubic lattice where each site $\mathbf{X}$ has b_m neighbors (figure 1). Each site is characterized by a particle distribution function $N_i(\mathbf{X},T)$ which evolves according to the Lattice Boltzmann Equation:

$$N_i(\mathbf{X}+\mathbf{c}_i, T+1) - N_i(\mathbf{X}+\mathbf{c}_i, T+1) = \Omega_i, \quad (1)$$

where T is the time variable, the index i indicates the neighbor, $\mathbf{c}_i$ is a velocity vector pointing to neighbor i ($i=0$ refers to the rest particle distribution). The term in the right side is called collision term and is writen in such way that

$$\sum_{i=0}^{b_m} \Omega_i = 0, \quad (2)$$

$$\sum_{i=0}^{b_m} \Omega_i \mathbf{c}_i = 0, \quad (3)$$

in order to preserve the mass and momentum of each site.

The evolution of the model, given by equation (1), can be split in two processes. In the first, designated as *collision*, the distribution function $N_i(\mathbf{X}, T)$ is changed by the action of the collision operator. In the second process, called *propagation*, the values N_i are propagated to the neighbor sites, in accordance with the direction of the vector $\mathbf{c}_i$.

The mass and momentum variables are defined with the help of the distribution function:

$$\sum_{i=1}^{b_m} N_i + N_0 = r, \quad (4)$$

$$\sum_{i=0}^{b_m} N_i \mathbf{c}_i = r\boldsymbol{u}\,. \tag{5}$$

The BGK Collision Term

In present paper a single-time-relaxation approximation for the collision is used (BGK model[6])

$$\Omega_i = \frac{(N_i^{eq} - N_i)}{t}, \tag{6}$$

where τ is the relaxation time and N_i^{eq}, the equilibrium distribution, is determined in order to obtain the desired macroscopic equations. The rate of change toward the equilibrium distribution is imposed to produce the viscosity of the fluid, which is the only macroscopic property related to collisions in single-fluid flows.

Equilibrium distributions

When equilibrium is reached collision process should not affect the particle distribution. Therefore, the equilibrium distributions N_i^{eq} must be specified by the collisional invariants (ρ,u). To the case of small **u**, it can be written:

$$N_i^{eq}(r,\mathbf{u}) = A_i + B_{ia}\mathbf{u}_a + D_{iab}\mathbf{u}_a\mathbf{u}_b \qquad i=1,2,\ldots,b_m \tag{7}$$

where b_m indicates the number of directions of the lattice.

There are some conditions that must be imposed to N_i^{eq} in order to determine A_i, B_{ia} and D_{iab}. First, it is imposed that any transformation that preserves the lattice and direction *i* must not affect the equilibrium distribution. Hence it can be saw that

$A_i = A$ (remember that when $|\mathbf{u}| = 0$, N_i^{eq} must be symmetric)

$$B_{ia} = B\mathbf{c}_{i\acute{a}} \tag{8}$$

$$D_{iab} = D_1\mathbf{c}_a\mathbf{c}_b + D_2 d_{ab}\,.$$

Rest particles equilibrium distributions must be independent of the direction **i**, and can be write,

$$N_0^{eq}(r,\mathbf{u}) = b_r A + D_3 d_{ab}\,, \tag{10}$$

where b_r reefers to the number of rest particles being a free parameter.

Obviously, the distributions N_i^{eq} and N_0^{eq} must be in accordance with equations (4) and (5):

$$\sum_{i=1}^{b_m} N_i^{eq} + N_0 = r\,, \tag{11}$$

$$\sum_{i=0}^{b_m} N_i^{eq}\mathbf{c}_i = r\boldsymbol{u}\,. \tag{12}$$

These constraints allow to determine *A*, *B* and D_3. Remained parameters D_1 and D_2 are chosen in order to obtain macroscopic hydrodinamic equations, avoiding non-physical lattice gas effects such as:

i) a velocity dependent pressure ,

ii) breaking of galilean invariance.

The equilibrium distribution for the rest particles is, finally, given by

$$N_0^{eq} = r\left(\frac{b_r}{b_r + b_m}\right) - \frac{r}{2}(\boldsymbol{u})^2\,, \tag{13}$$

and for moving particles along the main axes

$$N_i^{eq} = r\left(\frac{1}{br + bm}\right) + r\left(\frac{2}{b_m}\right)\mathbf{c}_i\cdot\boldsymbol{u} + r\left(\frac{3}{b_m}\right)(\mathbf{c}_i\cdot\boldsymbol{u})^2 - r\left(\frac{1}{b_m}\right)(\boldsymbol{u})^2\,, \tag{14}$$

and for particles moving along the diagonals $\left(|\mathbf{c}_i| = \sqrt{2}\right)$, N_i^{eq} is given by

$$N_i^{eq} = \frac{r}{2}\left(\frac{1}{br + bm}\right) + \frac{r}{2}\left(\frac{2}{b_m}\right)\mathbf{c}_i \cdot \boldsymbol{u} + \frac{r}{2}\left(\frac{3}{b_m}\right)(\mathbf{c}_i \cdot \boldsymbol{u})^2 - \frac{r}{2}\left(\frac{1}{b_m}\right)(\boldsymbol{u})^2. \quad (15)$$

Scaling to physical variables

Using h and ε as, respectively, a spatial and a time scale

$$\begin{aligned} \mathbf{x} &= \mathrm{h}\mathbf{X} \\ \mathrm{t} &= \delta \mathrm{T} \end{aligned} \quad (16)$$

x and t may be considered as physical variables, varying continuously in the spatial and time domain of the physical system to be described, when h and δ are small.

Hydrodynamic LB equations

Using Chapman-Enskog method in the Lattice-Boltzmann equation, in the limit of low Knudsen number Kn=h/L=δ/Γ<<1, where L is a characteristic length and Γ is a characteristic time and neglecting terms beyond second order, leads to

$$\partial_t(\rho) + \partial_\beta(\rho u_\beta) = 0 \quad (17)$$

$$\partial_t(r u_a) + \partial_b(r u_a u_b) = -\partial_a(p) + n\partial_b\left[\partial_b(r u_a) + \partial_a(r u_b)\right] \quad (18)$$

where the pressure p and the kinematic viscosity ν are given by

$$p = \frac{1}{3} r \quad (19)$$

$$\nu = \frac{1}{6}(2t - 1). \quad (20)$$

The mass balance equation is, exactly, the same equation obtained in classical hydrodynamics. Considering low Mach number (r constant), the obtained momentum balance equation will be, clearly, in agreement with the Navier-Stokes equation.

Boundary conditions

The non-slip condition near the walls can be easily obtained imposing the bouncing-back boundary condition, i. e., all particle that would hit the walls in the propagation step reverses its direction in this step. There are other (and more precise) ways to ensure non-slip condition[7], but the bouncing-back boundary condition was chosen due to its simplicity.

In the simulations periodic boundary conditions were always used. To simulate flows through porous media it was added a non-solid region in the inlet and outlet, and flow was forced adding some amount of momentum at each site of this region, according to the equation

$$N_i(\mathrm{X} + \mathbf{c}_i, \mathrm{T} + 1) - N_i(\mathrm{X} + \mathbf{c}_i, \mathrm{T} + 1) = \Omega_i + \mathbf{f} \cdot \mathbf{c}_i \quad (21)$$

where **f** indicates the direction of the force applied and is proportional to its magnitude.

3. Results

Poiseuille flow

A simulation was performed for the flow between two parallel plates in order to compare the results obtained using the Lattice-Boltzmann method and results predicted using the Navier-Stokes equations. As can be seen in figure 2 the simulated points fits exactly in a parabolic profile.

Two-dimensional flows

Several simulations of flows past circular cylinder were done. In figure 3 is shown the simulated flow and a photograph (from Batchelor[8]), the similarity of the images is great. In

figure 4 is presented three stages in the development of flow past circular cylinder, reveling the formation of the von Karman's vortex street. A flow through a simplified 2D porous media is presented in figure 5.

Intrinsic permeability

Three-dimensional representations of the porous structure, reconstructed from petrographic thin plates by using Liang *et al.* method[9][10] were used to calculate intrinsic permeability of several sandstones. Information about this reconstruction method can be found in a companion paper submitted to present meeting[11].

Table I gives some simulation results and comparison with experimental data for sandstones. The sampling factor *n* appearing in the table is an important reconstruction parameter Sampling factor n=1 means that three-dimensional representation and the binary image have the same spatial resolution. As sampling factor *n* increases, resolution decays with the same ratio. In general, it is very difficult to preserve original resolution in reconstructed representations, due to limitations inherent to the reconstruction method itself, which fails in preserving the fine details of the porous structure. In this sense, the best-reconstructed microstructure is considered to be the one generated with the sampling factor that gave the best agreement between pore size distributions, measured on the original binary image and on cross-sections of the three-dimensional representation. The size of the reconstructed structure is another important parameter. The size must be sufficient to ensure statistical homogeneity with respect to fluid flow problem[12]. Structures of 100^3, 120^3 and 150^3 where used to perform the simulations presented.

4 – Conclusions

In this work the Lattice Boltzmann is presented as tool for doing hydrodynamic simulations, especially when inertial terms are important. Several results are showed: simulations of two-dimensional flows and calculation of intrinsic permeability for several sandstones, in this case is important to mention that the results of intrinsic obtained by Lattice gas models[13] are better them the ones presented here. We suppose these results could be better if it was increased the image sizes used, but computational requirements rapidly limit this option.

Acknowledgements

The authors would like to acknowledge CENPES/PETROBRAS (Centro de Pesquisas e Desenvolvimento Leopoldo A. Miguez de Mello) for providing the images and experimental data for Berea and Brazilian sandstones and the financial support of CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), PADCT (Programa de Apoio ao Desenvolvimnto Científico e Tecnológico), RHAE (Programa de Formação de Recursos Humanos em Areas Estratégicas) and FINEP (Financiadora de Estudos e projetos).

References

[1] – Rothman, D. H., Zaleski, S. (1997). *Lattice-gas cellular automata: simple models of coplex hydrodynamics* (Cambridge University Press).

[2] – Frisch, U., Hasslacher B., Pomeau Y. (1986). "Lattice-Gas Automata for the Navier-Stokes Equation" *Physical Review Letters* **56**, 1505-1508.

[3] – Frisch, U., d'Humières, D., Hasslacher, B., Lallemand, P., Pomeau, Y., Rivet, J. (1987). "Lattice Gas Hydrodynamics in Two and Three Dimensions" *Complex Systems* **1**, 649-707.

[4] - McNamara G. G. and Zanetti G. (1988). "Use of the Boltzmann Equation to Simulate Lattice-Gas Automata" *Physical Review Letters* **61**, 2332-2335.

[5] – Higera, F. and Jimenez, J. (1989). "Boltzmann approach to lattce gas simulations." *Europhys. Lett.* **9**, 663-668.

[6] – Qian, Y. H., d'Humières D., Lallemand P. (1992) "Lattice BGK Models for Navier-Stokes Equation" *Europhys. Lett.* **17**, 479-484.

[7] – Zou, Q. and He, X. (1997) "On pressure and velocity boundary conditions for the lattice Boltzmann BGK model" *Phys. Fluids* **9**, 1591-1598.

[8] – Batchelor, G. K. (1967) *Fluid Dynamics* (Cambridge University Press).

[9] – Liang, Z.R.. "Computer Generation and Application of 3-D Reconstructed Porous Structures: From 2-D Images to the Prediction of Permeability" Doctoral Thesis, Depto de Eng. Mecânica, UFSC, 1997.

[10] – Liang, Z. R., Fernandes, C. P., Magnani, F. S. and Philippi, P. C. (1998) "A Reconstruction Technique of 3-D Porous Media by using Image Analysis and Using Fourier Transform", *Journal of Petroleum Science and Engineering***, 21**, 3-4, 273-283.

[11] – Damiani, M. C., Fernandes, C. P., Bueno, A. D., Santos, L. O. E., da Cunha Neto, J. A. B. and Philippi, P. C. "Predicting Physical Properties of Reservoir Rocks from the Microstructural Analysis of Petrographic Thin Sections". Submitted to Produccion 2000 / Aplicaciones de la ciencia en la ingeniería de petróleo**, May 08-12 /2000, Foz de Iguaçu**

[12] – van Genabeek, O. and Rothman, D. H. (1996) "Macroscopic Manifestations of Microscopic Flows Through Porous Media: Phenomenology from Simulation" *Annu. Rev. Earth Planet. Sci.* **24**, 63-87.

[13] –Santos, L. O. E., Philippi P. C. and Damiani, M. C. "Simulation of fluid flow inside porous structures by using lattice boltzmann model". Submitted to Produccion 2000 / Aplicaciones de la ciencia en la ingeniería de petróleo**, May 08-12 /2000, Foz de Iguaçu**

Figures and Tables

Sandstone sample	Experimental Permeability	Recontruted Image Size	Sampling Factor	Simulated Permeability
67409	441	100	5	743
67409	441	150	5	709
L2861	114	100	6	493
L2861	114	150	6	499
51440	316	100	5	458
51440	316	150	5	350
Berea	200	100	5	355
Berea	500	120	5	637

Table 1 – Intrinsic permeability results.

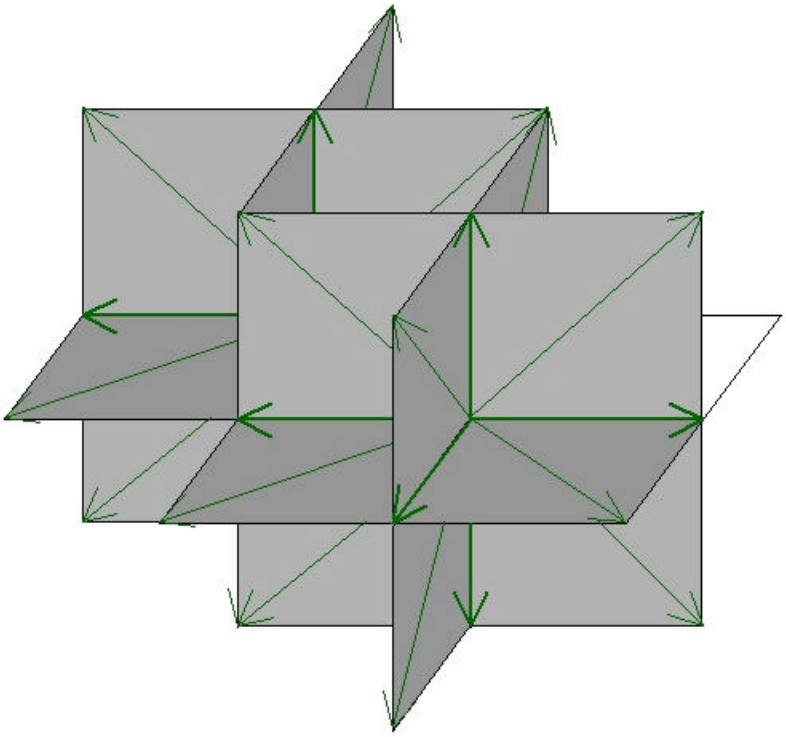

Figure 1. – Three dimensional lattice with 18 velocities.

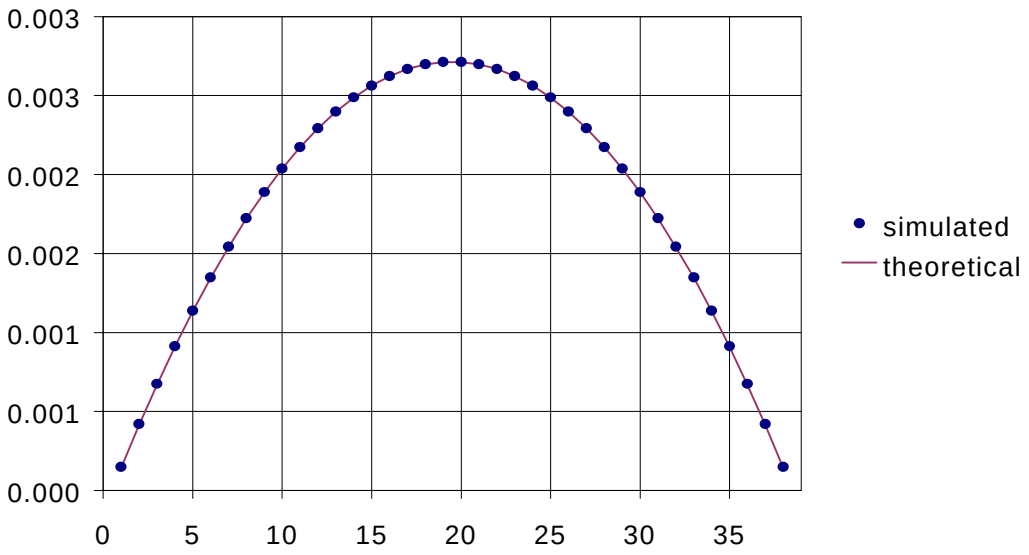

Figure 2. Velocity profile of a flow between two parallel plates. The simulation was done using a lattice of 40 sites in the direction perpendicular of flow. Periodic conditions were also used.

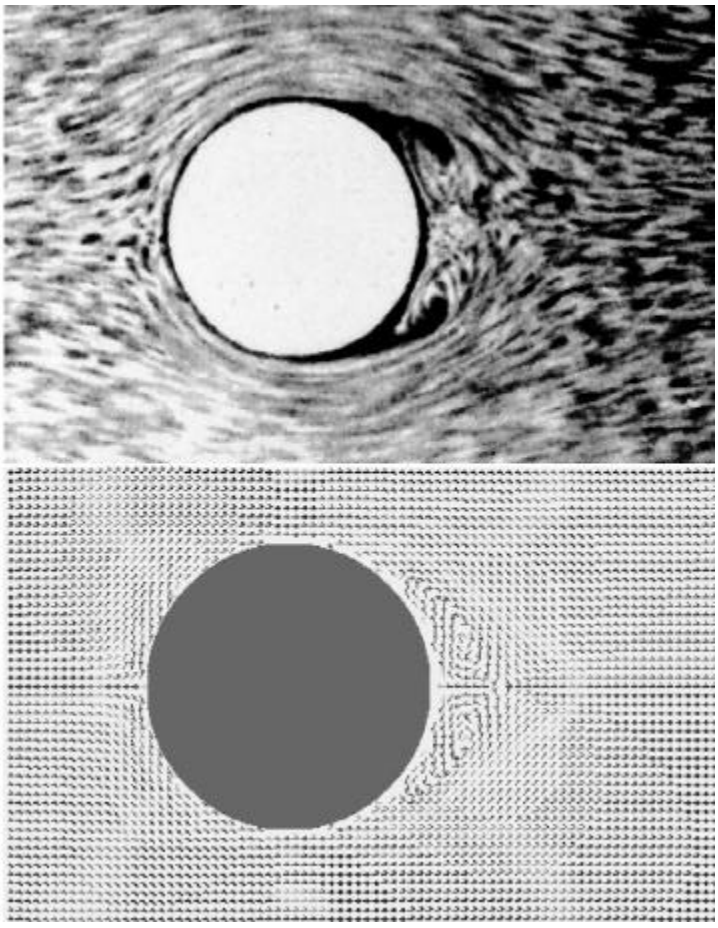

Figure 3 Simulation of a flow past a circular cylinder and a photograph of such flow.

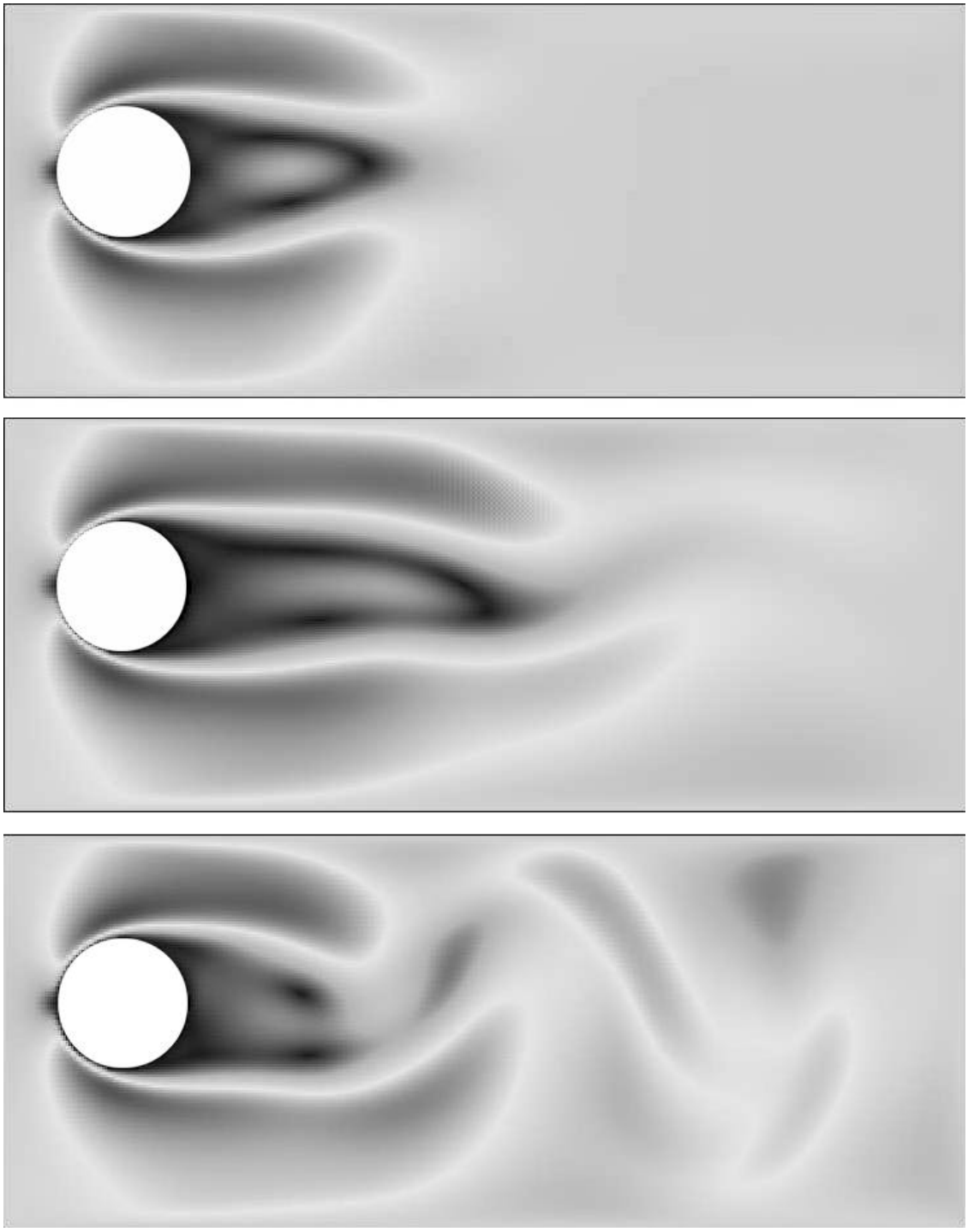

Figure 4. Three stages in the simulation of flow past a circular cilinder.

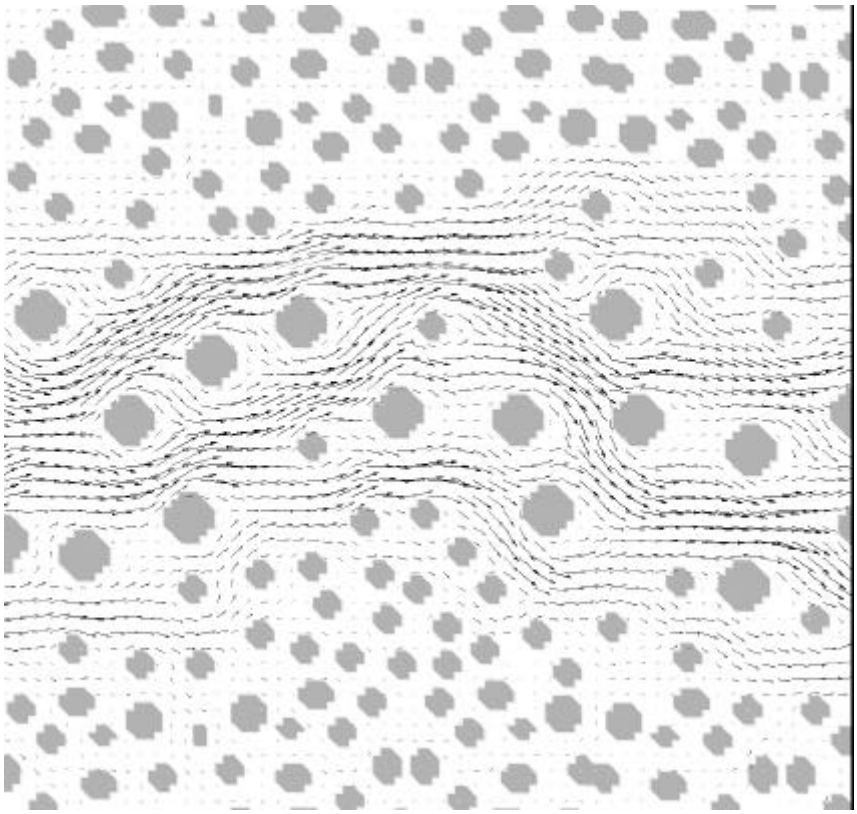

Figure 5. Velocity field of a throug a 2D porous media.