

AN INNOVATIVE DESIGN APPROACH TO REDUCE DRILL STRING FATIGUE WITH CASE HISTORIES HIGHLIGHTED

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ABSTRACT

Fatigue accounts for most drill string failures.^{1,2} The complexity of fatigue precludes prediction of a component's fatigue life in cycles to failure. This paper describes a new design tool called Curvature Index which is based on a comparative design methodology. Curvature Index normalizes unknown factors affecting fatigue, thereby providing a quantitative comparison of fatigue performance among several alternatives based on known factors. This paper presents the technology and one case history in which Curvature Index was used to improve the drill string fatigue design for a US Gulf Coast operator drilling a 6 1/2" hole which was plagued with fatigue failures. After the design improvement, the section was drilled to planned depth with no additional fatigue failures. The case presented illustrates how Curvature Index can be used to optimize fatigue design, thereby reducing failure frequency and associated costs.

DRILL STRING DESIGN

In designing a drill string, the designer rarely decides the attributes of a particular component. Instead, he or she forms a "string" by screwing together up to several hundred off-the-ground items. In this activity, the designer will try to choose components that balance a number of often-conflicting needs, including loads, hydraulics, hole cleaning, rate of penetration, steering, measurement and perhaps most critical, structural soundness of the drill string itself. In maintaining structural soundness, the designer faces two separate design challenges. First, preventing overload failure and second, preventing fatigue failure. The market for drill string components is mainly a rental market in which a single component will be used and reused a number of times by a number of designers. Furthermore, it is a market in which components are specified and purchased primarily for strength under high, relatively static loads. Resistance to overload failure, not resistance to fatigue, is the principal focus of standards and specifications covering drill stem components. Because of these two factors, the designer is often left in the dark on issues that are very important for affecting a workable *fatigue* design. This is best illustrated by looking at the fatigue mechanism and the approaches available for controlling it.

TWO PHASES OF FATIGUE

The fatigue life of a component can be divided into two phases: Crack initiation and crack propagation. The *initiation phase* begins when the component first goes into service in some cyclic stress environment, like rotary drilling. It ends when a fatigue crack is of sufficient size that further crack growth is relatively independent of component geometry and surface finish. The *propagation phase* is the service period that follows, during which the crack, unless detected by inspection, grows until the component fails by leak, brittle fracture or gross plastic deformation. The relative duration of the two phases can vary widely depending on the circumstances of service. Surface finish is very important at low stress amplitudes, and under these conditions, the majority of a component's fatigue

life is spent in crack initiation.³ Fatigue damage is cumulative from one job to the next, but no economically viable means are available to determine the extent of fatigue damage until a crack has formed and grown large enough to be found by inspection. So when a designer picks up a component, it will likely have accumulated some prior damage, the extent of which he or she has no way of knowing.

THE PROBLEMS: COMPLEXITY AND LACK OF SPECIFIC KNOWLEDGE

Several approaches to mitigating fatigue have been available for years, and many attempts have been made to apply them to drill strings. None of these attempts have yet achieved any widespread success. These approaches can be grouped into two broad categories: Those that cover the entire life of a component, and those that apply only to the crack propagation phase.

Stress or strain amplitude vs. fatigue life

These two views can be illustrated by “S-N” curves, an example of which is shown in Figure 1. The stress or strain amplitude on the y-axis refers to the point of greatest stress or strain amplitude on the component, not the bulk stress over the entire load-bearing area. The x-axis represents total fatigue life. This view of fatigue has the advantage of applying to both initiation and propagation phases, but has a major disadvantage in that, during the crack initiation phase, *point stress* is very sensitive to component geometry and surface condition. (This is why laboratory S-N fatigue tests are routinely done with polished test specimens.) Unfortunately, the multitude of stress concentrators that are intrinsic to drill string components such as internal upsets on drill pipe, thread roots in BHA connections and the many cuts, notches and pits that accumulate during service make determination of maximum *point* stress or strain impossible for the designer. Without this knowledge, there is no way to enter the plot and determine expected fatigue life. Another major disadvantage of this approach is its sensitivity to environmental corrosiveness, particularly at low stress/strain amplitudes. This is also illustrated in Figure 1, which shows the S-N performance of identical steel test specimens in benign and corrosive environments.⁴ Not only does a corrosive mud create more stress concentrators (corrosion pits) – it also accelerates crack initiation and growth which precludes the designer’s use of fatigue endurance limit as a design tool.⁵ A final disadvantage of the S-N approach is that the designer has no way of knowing the extent of accumulated fatigue when a rental component is picked up. In spite of its disadvantages however, the strain vs. cycle life approach can be very useful for comparing crack *initiation* behavior in components of different but known geometries, as discussed further below and in reference 6.

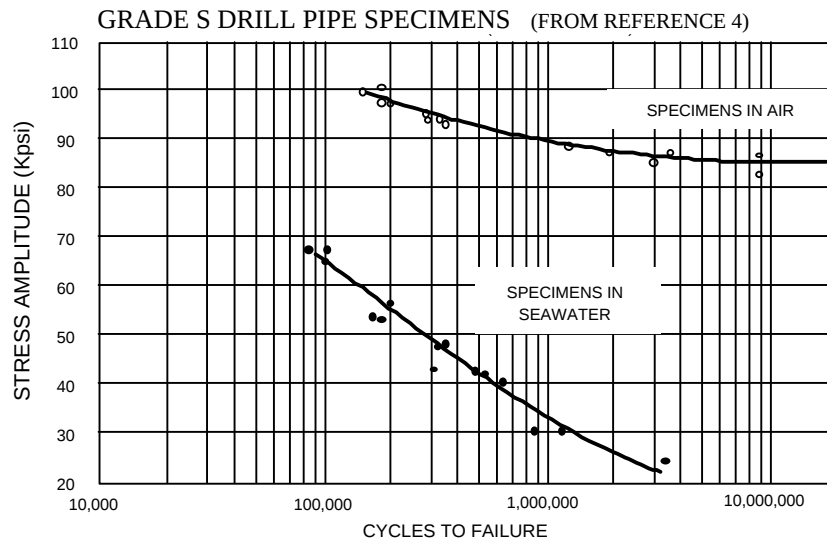


Figure 1 – S-N curve for grade S drill pipe test specimens

Stress intensity range vs. crack growth rate

The other general approach to fatigue design is illustrated by Figure 2. With this approach, the designer can negate many of the disadvantages of the S-N approach by assuming that a tiny crack at the threshold of detection is already present in every component. When a component goes into the designer's well, the growth per cycle of every hypothetical crack can be estimated based on the severity and duration of service that the component is experiencing in the hole. When any hypothetical crack has (hypothetically) grown enough to present a problem, the component can be taken out of service and inspected. Thus, any real crack that had indeed been present initially will be found before it causes a problem. If no crack is found, then none existed initially, and the component is placed back into service with a "re-zeroed" hypothetical crack. The use of this approach on drill strings was first suggested by Dale.⁷ Advantages of this view of fatigue are that component geometry and surface finish are much less relevant, since a crack has already formed (at least hypothetically). Thus, stress concentrators, like slip cuts and thread roots, no longer confuse the calculation of fatigue life. Past life and accumulated fatigue are largely irrelevant as well, since a component's life can be "restarted" by inspecting it. The disadvantage of this view is that no account can be made of component behavior in the crack initiation phase.

Either of two equations may be used to predict crack growth: The Paris equation,⁸ shown as Equation 1 below, or the Forman equation,⁹ shown in Equation 2. We consider the Forman equation superior for use as a design tool, as it takes into account the ratio of minimum stress to maximum stress (R).

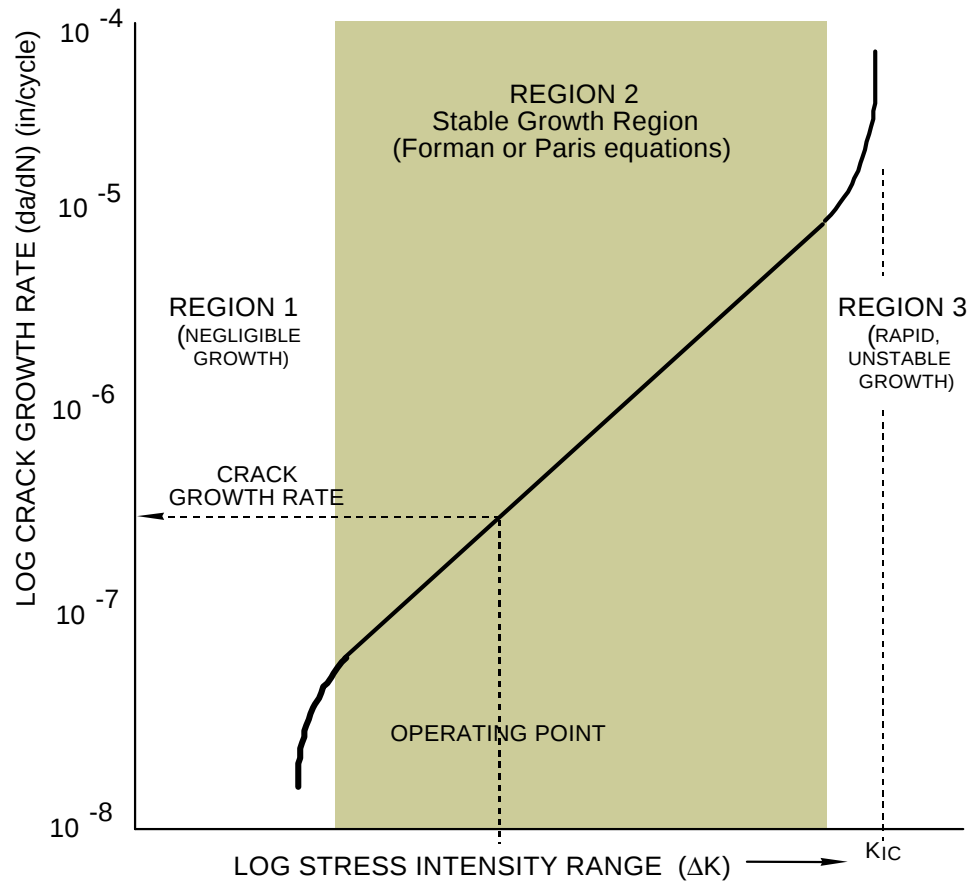


Figure 2 – Crack growth rate curve

$$\frac{da}{dN} = C\Delta K^n \quad (\text{Paris equation}) \dots\dots\dots(1)$$

$$\frac{da}{dN} = \frac{C\Delta K^n}{(1-R)K_{IC} - \Delta K} \quad (\text{Forman equation}) \dots\dots\dots(2)$$

Where:

- da/dN = Crack growth rate (in/cycle)
- ΔK = Stress intensity factor range ($ksi\sqrt{in}$)
- K_{IC} = Critical stress intensity factor ($ksi\sqrt{in}$)
- R = Stress ratio
- C = Crack growth equation coefficient
- n = Crack growth equation exponent

The crack growth rate vs. stress intensity range view is used with success in many fields, especially aerospace. However, material must be thoroughly (and expensively) tested to establish values for material constants C, n, and K_{IC}, inputs to the above equations. Also, the cost of a failure must be

great enough to justify extraordinary efforts to ensure that production material consistently duplicates tested material for these properties. Failure costs for drill strings, while high in dollar terms, do not approach failure costs in applications that involve risk to humans. Thus, values for the material constants C , n , and K_{IC} , which our designer needs to accurately estimate crack growth, do not exist for the vast majority of material that is already in the market and available for use. Neither do the economics of the rental drill string market justify the huge expenditure it would take to control these properties for newly manufactured material as it enters the market.

COMPARATIVE DESIGN OR “OTHER THINGS EQUAL”

Because of the issues discussed above, making an accurate forward estimate of absolute fatigue life in cycles to failure is not practical in everyday drill string design. The designer simply does not know all the necessary variables to plug into the formulas. Yet he or she does know *many* of them, and what is known can form the basis for quantitative *comparison* between alternatives. The approach introduced in this paper is called *comparative design*. The drill string designer’s job is essentially to choose between competing alternatives to balance many issues, only one of which is fatigue. Thus, the comparative design approach is to quantitatively express the *relative* fatigue performance of the alternatives the designer is likely to be considering, based only on variables the designer is likely to know or is able to assume with some certainty. This approach, while unable to tell absolutely how long a component will last in a given set of circumstances (because many factors are still unknown), will provide a quantitative estimate of how one alternative under consideration compares to another, *other things equal*. In this way, *relative* fatigue performance becomes one of the many criteria upon which one design alternative is preferred over another, very much like relative hydraulic performance in some circumstances might cause a designer to choose 6 5/8" drill pipe over 5" pipe.

To provide the data needed for this approach, we use the Forman Model to calculate normalized fatigue life for a variety of cases, varying the factors which the designer will know, and *holding constant the variables which the designer is not likely to know*. We express relative fatigue performance with the dimensionless index, *Curvature Index*, for drill pipe tubes. The derivation is given in Appendix A.

Normalized fatigue life

The fatigue life determined by the Forman crack growth model for a given component in a given set of circumstances is called *normalized fatigue life*. It is actually the period, expressed in stress cycles, that is required to grow an existing fatigue crack from an assumed size to failure. It is *normalized* in the sense that when we calculate crack propagation life for a different set of circumstances to compare against the first set, all the factors that the designer is *not likely to know* are held constant for both calculations, so the two sets of circumstances will vary only in ways that should be known to the designer. For example, consider a joint of drill pipe rotating in a dogleg. Several Forman model inputs are probably either known by the designer, easily calculated from known quantities, or may be assumed with reasonable certainty. These include:

Axial stress	Bending stress
Stress ratio (R)	Minimum stress intensity factor (K_{min})
Maximum stress intensity factor (K_{max})	Stress intensity factor range (ΔK)
Component geometry correction factor (F)	Crack shape correction factor (Q)
Maximum size of an existing crack (a)	

Other important factors however, will not be known with reasonable certainty, including:

- Forman crack growth coefficient (C)
- Forman crack growth exponent (n)
- Critical stress intensity factor (K_{IC})
- Effects of mud corrosiveness on crack growth rate

By running a series of calculations for normalized fatigue life under a variety of circumstances, varying those factors in the first group and holding constant those in the second, it is possible to make meaningful comparisons between design alternatives. The remaining task is to express the results in terms both understandable and usable by the drill string designer.

Curvature Index (CI)

For drill pipe tubes, translating calculation results into usable terms is done with Curvature Index (CI) plots, examples of which are shown in Figures 3 and 4. Curvature Index is a dimensionless number defined by Equation 3.

Because it is simply a restatement of Forman fatigue life from one set of circumstances to another, CI enables the designer to make quantitative fatigue life comparisons between two or more sets of circumstances. Any two sets of circumstances with the same CI would be expected to have the same rate of fatigue damage per cycle, and the ratio of Curvature Indices represents the degree that two sets of circumstances will vary one from another, *unknown variables equal*. For example, Figure 3 shows that rotating 5" 19.50# S-135 premium class pipe in a 7 °/100 ft dogleg with 210 kips tension is equivalent to rotating the same pipe in a 3 °/100 ft dogleg with 440 kips tension. Furthermore, both would grow existing fatigue cracks at twice the rate of 5 7/8" 23.40# G-105 premium class pipe in a 5 °/100 ft dogleg with 215 kips tension beneath (Figure 4).

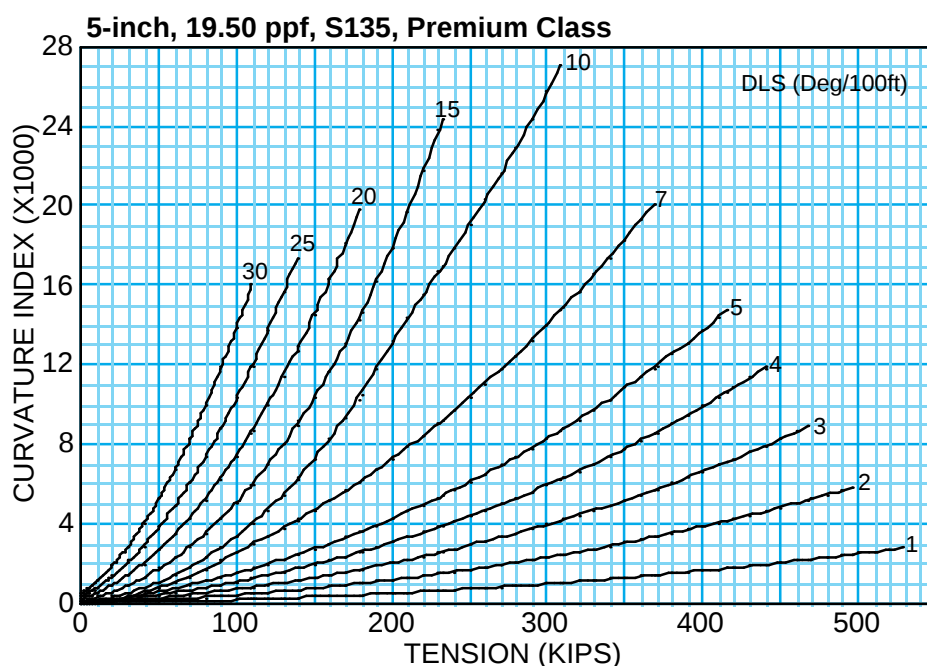


Figure 3
Curvature Index for 5-inch, 19.50 ppf, S135, Premium Class Drill Pipe

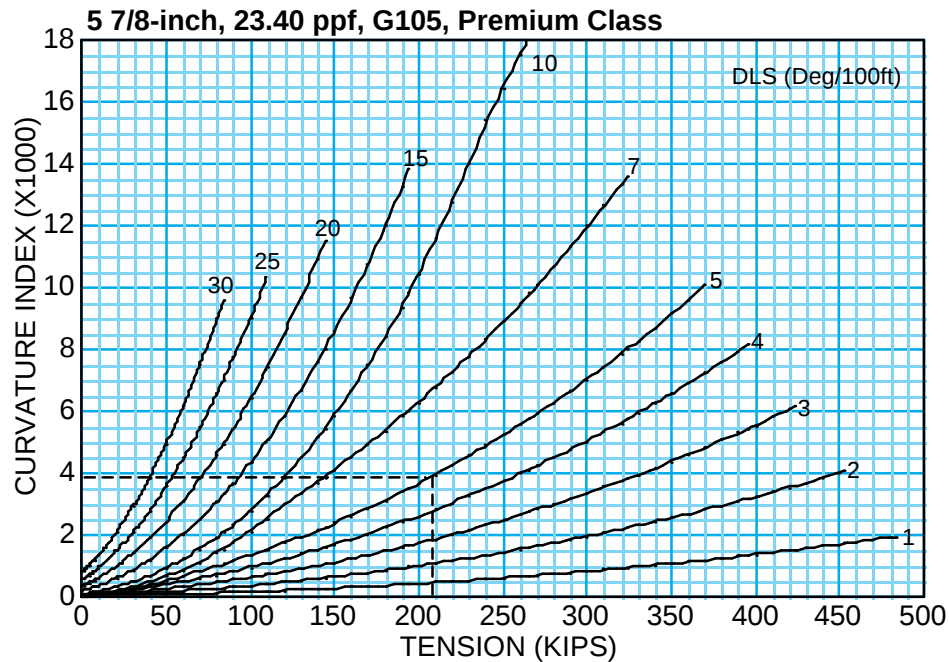


Figure 4 - Curvature Index for 5 7/8-inch, 23.40 ppf, G105, Premium Class Drill Pipe

$$CI = \frac{5 \times 10^8}{\text{number of cycles to failure}} \dots\dots\dots(3)$$

Availability of other curves

The calculation effort to determine Curvature Index for a number of design alternatives is substantial. Therefore, we have pre-calculated Curvature Index covering 183 cases for normal weight and thick-walled drill pipe. The results are reduced to curves similar to those in this paper, and are available for design purposes in DS-1™ Third Edition.

Advanced use of the concept

No tools other than charts are required for the designer to manually apply comparative fatigue design methods. However, suppose our designer did know the material constants necessary, and wanted to track fatigue crack propagation according to the method proposed by Dale.⁷ As Dale pointed out, crack growth rate on any component in the drill string will vary widely with the characteristics of the hole, the location of the component at any moment, and the time and speed at which it rotates at every location. Therefore, component identity must be maintained and fatigue history accumulated by component or component group as each passes through the various curves and straight sections in each hole. Making sense of all this seems dauntingly complicated, yet software programs that will accomplish it are available. Means for keeping identity in these programs can be as simple as

manually entering a component's serial number. This level of attention will require more involvement and effort than using the comparative design charts above, but may be justified in critical cases.

Caution: Other things are not always equal

Fatigue crack initiation will often take longer than crack propagation, and the geometry and surface finish of any component can play huge roles during crack initiation. The reader is reminded that the approach given in this paper is much simplified in that it applies only during the propagation phase. Yet many choices our designer makes will significantly affect fatigue behavior during the initiation phase. Thus, the designer's decisions will help determine whether all those cracks being analyzed by the Forman model remain purely hypothetical or become all too real. What these design choices will have in common is that the alternatives will differ in some known and predictable geometrical way. For example, reference 6 shows that 6 5/8 Regular connections in 8" drill collars will form cracks and fail far more rapidly than will NC56 connections operating under identical conditions. The reason cited is that the 6 5/8 Regular geometry promotes crack initiation more readily than the NC56, due to differences in taper and thread form. So even though a crack, once formed, will grow about as quickly in either connection, the NC56 is a far better choice because it performs better in the crack initiation phase. Reference 6 uses the same approach to establish optimum dimensions for stress relief grooves in BHA pins, and reference 10 applies this rationale to estimate how slip cuts of various depths promote crack initiation. Since component geometry only figures prominently during the initiation phase, an S-N approach called the Morrow Strain Life Model, described in [6] and [12], is used to compare components having different geometries.

A CASE STUDY

In 2003, during the drilling of an exploration well in the gulf coast region, five drill string fatigue failures occurred within a six-day period. At one point, a tube that was recently inspected washed out after drilling only 9 1/2 hours. All of the failures occurred in the same shallow depth interval in the 8 1/2" hole section.

Investigation showed no metallurgical problems with the drill string and no apparent issue with the trajectory, as the MWD survey data (taken at 100 ft intervals) showed a dogleg severity less than 1 °/100 ft over the interval of interest. However, because of the number of failures in the same location, the operator ran a gyro survey through the problem interval at 5 ft stations. This survey showed a dogleg of approximately 20 °/100 ft in the interval where the failures were occurring. Fatigue was clearly the failure mechanism, and because of the probability of more fatigue failures, the economic feasibility of continued drilling was cast into doubt.

Designing a new fatigue-resistant drill string became the focus of the investigation. When attempting to mitigate fatigue damage, four main items should be addressed:

- Mean Tensile Stress - Simply stated, the higher the mean tensile stress in a component that is also undergoing cyclic stress excursions, the shorter its fatigue life will be.
- Stress Amplitude - Higher stress amplitude equates to a shorter fatigue life, all else equal.
- Corrosiveness - The more corrosive a mud system, the shorter the fatigue life of a component in that system will be, other things equal.

- Fracture Toughness - The tougher the material, the more difficult it is for a crack to grow within a given component.

With the above factors in mind, the fatigue resistance of each of the six drill string configurations shown in Table 1 was evaluated. Designs 1 and 2 were two of those that suffered the fatigue failures. Designs 3, 4, 5 and 6 were new designs that were analyzed for possible use (Design 3 was ultimately selected).

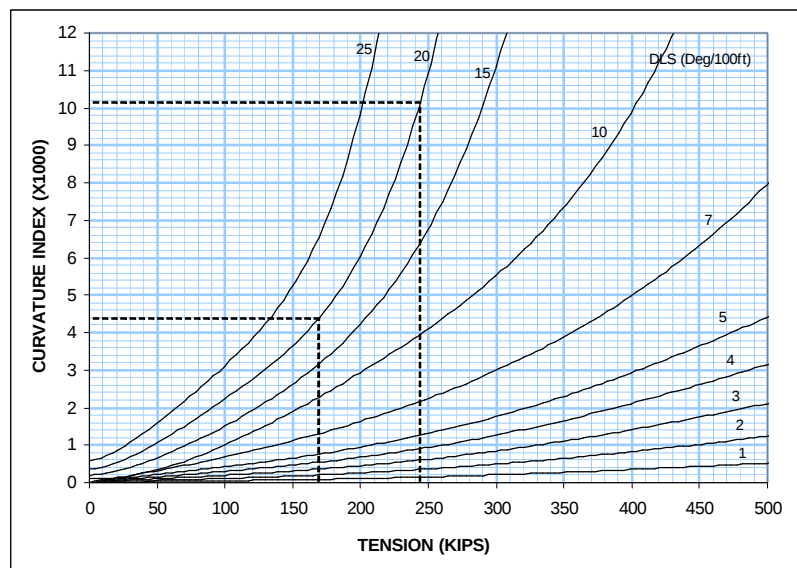
Design	5" HWDP	5 1/2" HWDP	5" 19.5#	3 1/2" 13.3#
1	x	x	12,000'	3,000'
2	1,000'	x	11,000'	3,000'
3	1,000'	x	100'	13,900'
4	x	1,000	100'	13,900'
5	x	x	x	15,000'
6	x	x	1,000'	14,000'

All the analyses were performed assuming the 20 °/100 ft dogleg was the primary cause of the fatigue failures. Since Curvature Index incorporates several of the factors that contribute to fatigue, it was used as the primary tool for choosing between design alternatives. Table 2 displays some design values for the two drill strings that sustained failures (Designs 1 and 2) and for the four drill string alternatives being considered (Designs 3, 4, 5 and 6).

Design	Side Load (Lbs/ft)	Pump Pressure (psi)	Curvature Index™	Tension below Dogleg (lbs)
1	470	2,700	29,700	221,000
2	500	2,800	10,100	243,000
3	350	3,900	4,400	168,000
4	350	3,900	5,700	169,000
5	320	4,200	31,400	153,000
6	330	3,900	15,700	157,000

(Side loads and tension below the dogleg are for rotary drilling with 10k WOB. Surface Pressure is at 180 gpm pump rate).

Design 3 was chosen because it had the lowest Curvature Index and hence the longest fatigue life. Compared to designs 1 and 2 which failed repeatedly, the expected life of Design 3 is some 230 percent longer. Figure 5 compares the Curvature Indices of Designs 2 and 3. Equal Curvature Index values represent equivalent fatigue life, and the ratio of Curvature Indices quantitatively represents the ratio of expected fatigue lives.



**FIGURE 5 -
Curvature Index for
5-inch, 49.3 ppf, 55-
ksi,**

80% RBW, HWDP.

In addition to redesigning the drill string, the operational parameters, drill string attributes and inspection protocol were addressed to further reduce the rate of fatigue damage accumulation and to help ensure the quality of pipe delivered to the rig.

Operations

- **Rotation.** Drill string rotation was minimized by running a downhole motor and reducing the rotary speed from 70 rpm to 20 rpm (the minimum rotary speed of the top drive). The reduction in rotary speed did not dramatically affect the rate of penetration. Decreasing the number of revolutions reduced the number of stress cycles, which minimized fatigue damage and increased the relative fatigue life by another 350 percent.
- **WOB.** Using a torque and drag program, a bit weight was selected to minimize the hook load through the dogleg while rotary drilling, without exceeding the drill pipe's critical buckling limit.
- **Mud Weight.** Maintaining the heaviest practical mud weight (considering fracture gradient, differential sticking and ECD) reduced the buoyed weight of the string and therefore reduced tension through the problem dogleg.

Drill String Attributes

Internal plastic coating works as a barrier between the corrosive mud and the ID of the drill pipe. Stress relief features in HWDP eliminate unengaged thread roots (stress concentrations) and allow more flexibility within the connection. Both of these features were incorporated for pipe run in the new design.

Other Attributes

In many circumstances, Charpy V-notch impact test results are excellent tools for estimating a material's ability to resist fatigue crack propagation. In addition, HWDP with a minimum yield strength of 100 ksi, compared to standard 55 ksi HWDP, will operate at a lower percentage of its yield strength given the same applied axial tension. This results in slightly better fatigue performance. Also, 100 ksi HWDP will have been quenched and tempered rather than normalized. This will provide a more refined and uniform grain structure, which generally yields better material fracture toughness. In this case however, neither test records nor high-strength HWDP were available in the short time at hand.

Inspection Program

DS-1™ Category 3-5 inspection³ with a Magnetic Particle Inspection of the slip and upset area was selected for the HWDP.

Summary

The remaining 2,000 feet to total depth was drilled with Design 3 over 17 days without another fatigue failure. This design, based on using Curvature Index as a comparative tool, was several times better with respect to fatigue when compared with the original designs that failed. The reduction in

rotary speed further increased the fatigue life of drill string. The additional steps of employing inspected pipe with internal plastic coating and stress relief features no doubt helped as well, though the exact amount cannot be quantified.

CONCLUSIONS

1. The calculation of absolute fatigue life is not practical for a drill string component owing to the complexity of the fatigue mechanism and the number of unknowns facing the drill string designer.
2. A dimensionless index, Curvature Index, has been developed to allow drill string designers to quantitatively compare design alternatives on the basis of normalized fatigue crack propagation life, for the purpose of selecting the one with the best fatigue performance.
3. Curvature Index will not give meaningful values of absolute fatigue life, but will provide a quantitative comparison of the relative fatigue lives of components operating under different sets of circumstances, unknown factors equal.
4. Curves that the designer can use for these comparisons are available in Volume 2 of DEA 74, DS-1TM Drill String Design and Operation.

ACKNOWLEDGMENTS

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APPENDIX A - FORMULAS

The Forman Crack Growth equation (A.1), from reference 9, was used for determining Curvature Index and Stability Index. The equation relates crack growth rate (da/dN) to stress intensity factor range (ΔK). The equation is an empirical formula in which values for constants “C”, “ K_{IC} ” and “n” are back-calculated from experimental measurements of da/dN vs. ΔK for the material in question. The formula can be used to predict the number of stress reversals that will grow an existing crack in the subject material from a given size to failure.

$$\frac{da}{dN} = \frac{C \Delta K^n}{(1-R) K_{IC} - \Delta K} \quad (\text{for } \Delta K > 0) \dots\dots\dots (A.1)$$

$$\frac{da}{dN} = 0 \quad (\text{for } \Delta K \leq 0)$$

$$\Delta K = K_{\max} - K_{\min} \quad (\text{for } R > 0) \dots\dots\dots (A.2)$$

$$\Delta K = K_{\max} \quad (\text{for } R \leq 0) \dots\dots\dots (A.3)$$

$$R = \frac{s_a - s_b}{s_a + s_b} \dots\dots\dots (A.4)$$

$$K_{\max} = (s_a + s_b) \sqrt{\frac{p a}{Q}} F \dots\dots\dots (A.5)$$

$$K_{\min} = (s_a - s_b) \sqrt{\frac{p a}{Q}} F \dots\dots\dots (A.6)$$

Where:

da/dN = Crack growth rate, (in/cycle)

ΔK = Stress intensity factor range, (ksi $\sqrt{\text{in}}$)

$K_{\max}$ = Maximum stress intensity factor, (ksi $\sqrt{\text{in}}$)

$K_{\min}$ = Minimum stress intensity factor, (ksi $\sqrt{\text{in}}$)

K_{IC} = Critical stress intensity factor (material property), (ksi $\sqrt{\text{in}}$)

R = Stress ratio

C = Forman equation coefficient (determined empirically)

n = Forman equation exponent (determined empirically)

a = Crack depth, (in)

σ_a = Axial stress (ksi)

σ_b = Bending stress (ksi)

F = Component geometry correction factor

Q = Crack shape correction factor

When using the Forman Crack Growth Model to calculate crack growth rate, an initial crack size is assumed and used to calculate the stress intensity factor range for the first cycle. The crack growth increment for that cycle is calculated, and the crack growth increment is added to the previous crack size to obtain the new crack size. This process is repeated until the crack has grown enough so that the critical stress intensity factor is reached.

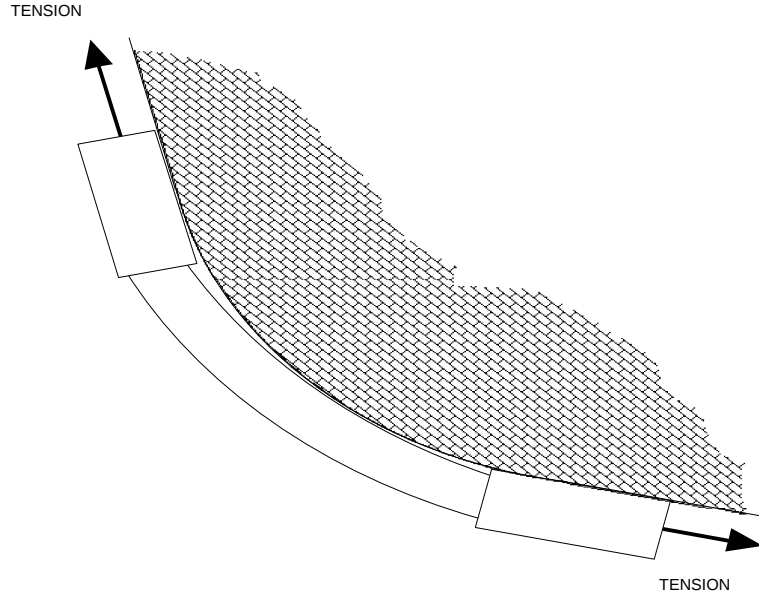


Figure A.1 – Joint of drill pipe in a curved hole section

Curvature Index

Consider figure A.1, which shows a drill pipe tube rotating in a dogleg while it's in simultaneous tension. Curvature Index is determined by first calculating the stress in the outer fiber of the drill pipe tube caused by bending (σ_b) as it rotates in a dogleg. This calculation is based in part on the work of Arthur Lubinski.¹¹ Equations A.9 and A.10 were obtained from Lubinski's work; however, the forms of these equations were derived to suit this application. Equation A.9 is used to test whether or not contact is occurring between the drill pipe tube and the hole wall for a given hole curvature and axial tensile load. Equation A.10 is used to calculate M_o for cases in which wall contact does not occur between the drill pipe tube and the hole wall. In the case of wall contact, equation A.10 will not apply. Therefore, it was necessary to derive equation A.11 to handle the wall contact case. This derivation was assisted by the work of Wu,¹³ who solved a similar problem for pipe under compressive loads. Since including the derivations of these equations here may confuse the reader who is trying to follow the calculation of Curvature Index, the derivations for equations A.9, A.10 and A.11 are given separately in Appendix B.

$$s_b = \frac{D}{2I} M_o \dots\dots\dots (A.7)$$

Calculate c :

$$c = \frac{1}{R_c} \dots\dots\dots (A.8)$$

Calculate c_c :

$$c_c = \frac{D_{TJ} - D}{L^2} \frac{(KL) \sinh(KL)}{2 - 2 \cosh(KL) + (KL) \sinh(KL)} + \frac{w_b L^2 \sin(q)}{EI (KL)^2} \dots\dots\dots (A.9)$$

If c is less than c_c , then the pipe does not contact the hole wall and M_o is given by equation A.10. If c is greater than or equal to c_c , then the pipe does contact the hole wall and M_o is given by equation A.11.

$$M_o = \frac{KL}{\tanh(KL)} \left[E I c - \frac{w_b L^2 \sin(q)}{(KL)^2} \right] + \frac{w_b L^2 \sin(q)}{(KL)^2} \dots\dots\dots (A.10)$$

$$M_o = \frac{w_b L^2 \sin(q)}{(KL)^2} + \frac{(KL/2)}{\tanh(KL/2)} \left[E I c - \frac{w_b L^2 \sin(q)}{(KL)^2} \right] + \frac{2 \cdot (KL/2)^2 \tanh(KL/2)}{(KL/2) - \tanh(KL/2)} \frac{EI \cdot r_c}{L^2} \dots\dots\dots (A.11)$$

$$K = \sqrt{\frac{T}{EI}} \dots\dots\dots (A.12)$$

$$r_c = \frac{D_{TJ} - D}{2} \dots\dots\dots (A.13)$$

Next, the axial stress in the drill pipe tube is calculated.

$$s_a = \frac{T}{A} \dots\dots\dots (A.14)$$

$$A = 0.7854 (D^2 - d^2) \dots\dots\dots (A.15)$$

The calculated axial and bending stresses are input into the Forman Crack Growth model to obtain the number of cycles to failure. The number of cycles to failure is then converted to Curvature Index using equation A.16.

$$CI = \frac{5 \times 10^8}{\text{number of cycles to failure}} \dots\dots\dots (A.16)$$

For detailed formula derivations, refer to SPE/IADC 87188

Nomenclature for CI calculations:

- A = Drill pipe tube cross sectional area, (in²)
- CI = Curvature Index
- D = Drill pipe tube outer diameter, (in)
- D_{TJ} = Drill pipe tool joint outer diameter, (in)
- d = Drill pipe tube inner diameter, (in)
- E = Young's modulus, (psi)
- I = Moment of inertia of drill pipe tube, (in⁴)
- L = Half the drill pipe tube length, (in)
- M_o = Bending moment on the drill pipe tube at the tool joint, (in-lbs)
- θ = Average inclination angle across the drill pipe tube, (radians)
- T = Axial tensile load, (lbs)
- R_c = Radius of curvature of hole wall, (in)
- c = Curvature of hole wall, (in⁻¹)
- c_c = Critical curvature of hole wall, (in⁻¹) (hole wall curvature required for the middle of the drill pipe tube to just contact the hole wall for a given axial tensile load)
- w_b = Buoyed weight per unit length, (lb/in)
- σ_a = Axial stress, (psi)
- σ_b = Bending stress, (psi)