JOINT SIMULTANEOUS INVERSION OF PP AND PS ANGLE GATHERS

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RESUMEN

Inversión simultánea conjunta de gathers de ángulos de ondas PP y PS
Presentamos una nueva aproximación a la inversión simultánea conjunta de gathers de ángulos de ondas PP y PS para la estimación de impedancia-P, impedancia-S y densidad. Nuestro algoritmo está basado en tres supuestos. El primero es que se mantiene la aproximación de linealidad para la reflectividad. La segunda es que la reflectividad PP y PS como función del ángulo será expresada por las ecuaciones linealizadas de Aki-Richards (Aki y Richards, 2002). La tercera es que existe una relación lineal entre el algoritmo de impedancia-P con la impedancia-S y con la densidad. Dados estos tres supuestos, se muestra como se puede encontrar una estimación final de impedancia-P, impedancia-S y la densidad perturbando un modelo inicial de impedancia-P. Después de una descripción del algoritmo, aplicamos nuestro método tanto a un modelo como un set de datos reales.

INTRODUCTION

Much inversion practice involves independent estimation of reflectivity followed by inversion to impedance. In this article we will discuss a type of inversion which is aimed at extracting P-impedance, S-impedance, and density from pre-stack *PP* and *PS* gathers. The objective of “simultaneous” inversion is to invert from multiple seismic traces to multiple inverted results. We also include some form of coupling between the variables, which should add stability to a problem that is sensitive to noise and very non-unique. This problem has been discussed by Margrave *et al.* (2001), but we present a new approach that allows us to invert directly for P-impedance ($Z_P = \rho V_P$), S-impedance ($Z_S = \rho V_S$), and density through a small reflectivity approximation similar to that of Buland and Omre (2003), and using constraints similar to those used by Simmons and Backus (1996). It is also our goal to extend an earlier post-stack impedance inversion method (Russell and Hampson, 1991) so that this method can be seen as a generalization to pre-stack inversion. The input data used to extract impedance and density consists of PP and PS angle gathers, so we refer to the method as “joint simultaneous” inversion.

THEORY

We start with the Fatti and Smith (Fatti *et al.* 1994) reformulation of the Aki-Richards linearized equation:

$$R_{PP}(\theta) = c_1 R_{P0} + c_2 R_{S0} + c_3 R_D, \tag{1}$$

where $c_1 = 1 + \tan^2\theta$, $c_2 = -8(V_S/V_P)^2 \sin^2\theta$, $c_3 = 4(V_S/V_P)^2 \sin^2\theta - \tan^2\theta$,

$$R_{P0} = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right], R_{S0} = \frac{1}{2}\left[\frac{\Delta V_S}{V_S} + \frac{\Delta\rho}{\rho}\right], R_D = \frac{\Delta\rho}{\rho}, \rho = \frac{\rho_2 + \rho_1}{2}, \Delta\rho = \rho_2 - \rho_1,$$

$$V_P = \frac{V_{P2} + V_{P1}}{2}, \Delta V_P = V_{P2} - V_{P1}, V_S = \frac{V_{S2} + V_{S1}}{2}, \Delta V_S = V_{S2} - V_{S1} \text{ and } \theta = \frac{\theta_1 + \theta_2}{2}.$$

Similarly to the Fatti equation, we can write down a linearized expression for the PS reflectivity in the following form:

$$R_{PS}(\theta) = c_4 R_{S0} + c_5 R_D, \tag{2}$$

where $c_4 = 4\tan\phi\left[\frac{V_S}{V_P}\sin^2\theta - \cos\theta\cos\phi\right]$, $c_5 = -\tan\phi\left[\frac{V_P}{2V_S} + \frac{V_S}{V_P}\sin^2\theta - \cos\theta\cos\phi\right]$,

$$\phi = \sin^{-1}\left(\frac{V_S}{V_P}\sin\theta\right) \text{ and } R_{S0} = \frac{1}{2}\left[\frac{\Delta V_S}{V_S} + \frac{\Delta\rho}{\rho}\right].$$

We also use the "small reflectivity" approximation to relate the impedances (Z) to the reflectivities (R), or

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} = \frac{\Delta Z_i}{2Z_i} \approx \frac{\Delta \ln Z_i}{2} = \frac{1}{2}\left[L_{i+1} - L_i\right], \tag{3}$$

where $L_i = \ln Z_i$. This applies to all three reflectivities. If we add the effect of the wavelet, we can express the seismic trace in matrix notation as $S = WDL$, where S is the n sample seismic trace in vector form, W is the wavelet in convolutional matrix form, D is the first-order derivative matrix and L is the n sample logarithm of reflectivities. However, the above equations still have not been coupled in any way. We can relate Z_S and ρ to Z_P using the following two linear equations:

$$L_S = kL_P + k_c + \Delta L_S, \quad \text{and} \tag{4}$$

$$L_D = mL_P + m_c + \Delta L_D, \tag{5}$$

where ΔL_S and ΔL_D are deviations away from the linear trend, as shown in Figure 1.

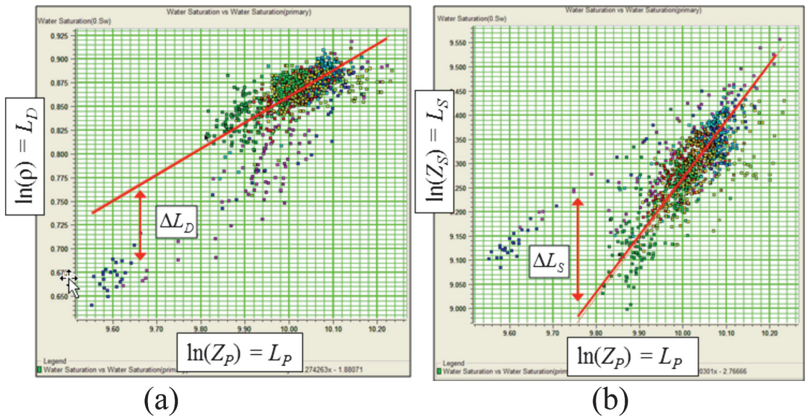


Figure 1. Linear fits between (a) the logarithms of P-impedance and density, and (b) of P-impedance and S-impedance, where difference terms, represent fluid anomalies.

This gives us the two sets of equations given by

$$S_{PP}(\theta) = \tilde{c}_1 W(\theta) D L_P + c_2 W(\theta) D \Delta L_S + 2c_3 W(\theta) D \Delta L_D, \qquad (6)$$

where $\tilde{c}_1 = c_1 + kc_2 + mc_3$, and

$$S_{PS}(\phi) = \tilde{c}_4 W(\phi) D L_P + (c_4/2) W(\phi) D L_S + c_5 W(\phi) D L_D, \qquad (7)$$

where $\tilde{c}_4 = kc_4/2 + mc_5$. The above equations can be written as a set of linearized and coupled equations which can be inverted using the conjugate gradient algorithm. Next, we will use model and real data examples to illustrate the inversion procedure.

MODEL EXAMPLE

To test the algorithm, a synthetic dataset was calculated using the Aki-Richards equation. Traces are modeled from 0 to 45 degrees. Figure 2 shows the well logs and P-wave and S-wave synthetics, where the *PS* data has been converted to *PP* time. The wavelet is a band pass filter 5/10-50/60 Hz.

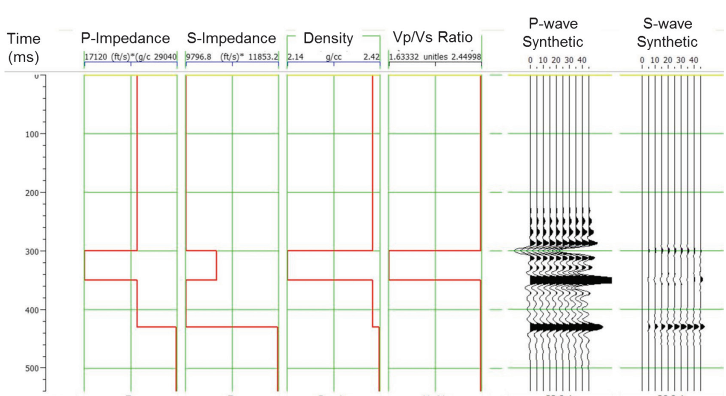
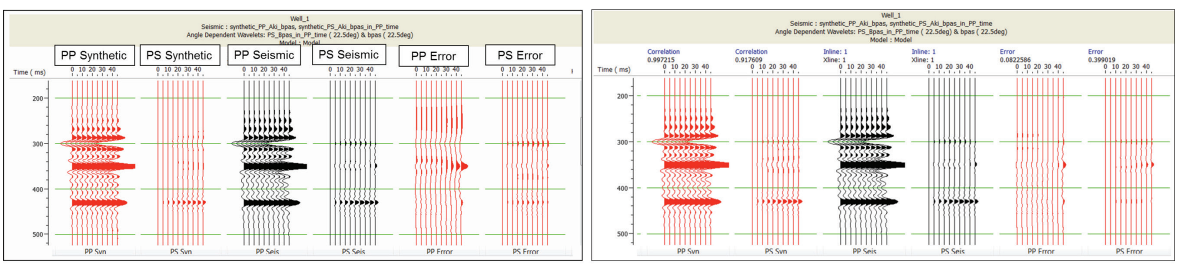


Figure 2. The P-wave and S-wave synthetics, where the PS data has been converted to PP time.

In the implementation just described, there are three issues which can affect the inversion result: constant γ versus variable γ, where $\gamma = V_S / V_P$, whether the PS data has been transformed to PP time or left in the original PS time, and the fact the angle θ is taken as the incident angle, when actually it should be the average between the incident and transmitted angles. The motivation for these approximations is that they allow the “c’s” in the Aki-Richards equation to be constant with time, which speeds up and stabilizes the calculation. We implemented both approaches to the inversion and compare them next. First, a time-variant gamma is calculated from the initial guess model and used to start the inversion. Next, every 5 iterations during the inversion process, the time-variant gamma is updated with the current values from the inversion. Finally, the correct average angle is used for all calculations, using the current time-variant velocity.

Figure 3a shows the synthetic error after 50 iterations using a fixed γ value of 0.5. The PS data has been transformed to PP time. Note the size of the error. Figure 3b shows the synthetic error after 50 iterations using a variable value of γ. Again, the PS data has been transformed to PP time. Note the decrease in the error when compared to the fixed value of gamma in Figure 3a.



(a) (b)

Figure 3. The synthetic error after (a) 50 iterations using a fixed γ value of 0.5, and (b) after 50 iterations using a variable γ value.

REAL DATA EXAMPLE

We will now apply this method to a real data example, consisting of a shallow Createaceous gas sand from Alberta. In this example only the PP and PS stacks were available. Figure 4 shows the correlation results between the well and the seismic data, with the well logs on the left and the PP stack on the right, in PP time.

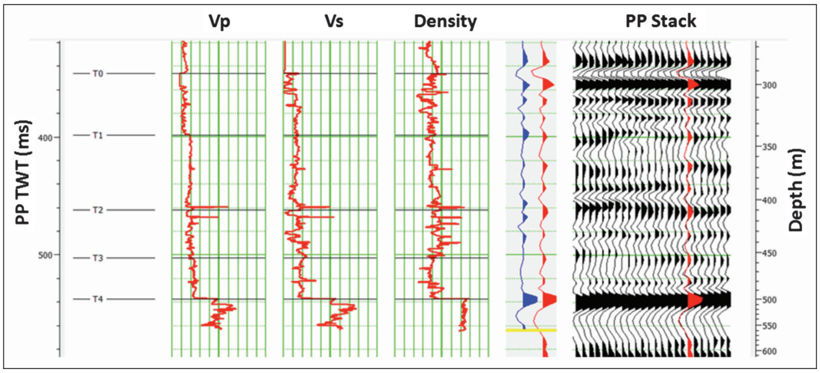


Figure 4. The correlation results showing the logs on the left and the *PP* stack on the right, in *PP* time.

Figure 5 shows the correlation results with the logs on the left and the *PS* stack on the right, in PS time.

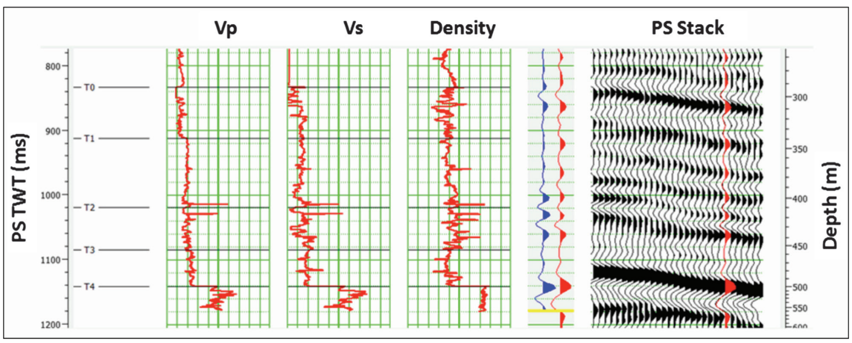


Figure 5. The correlation results with the logs on the left and the *PS* stack on the right, in *PS* time.

In total, four different inversions were run to test the parameters described for the synthetic data: a variable background gamma (V_S/V_P) with the *PS* data transformed to *PP* time, a variable background gamma (V_S/V_P) with the *PS* data in its original *PS* time, a constant background gamma ($V_S/V_P = 0.6$), with the *PS* data in PP time, and a constant background gamma ($V_S/V_P = 0.6$), with the *PS* data in *PS* time.

Table 1 shows the errors between the original and predicted logs for each test. Note in Table 1 that performing the inversion with the *PS* data in *PS* time is better than transforming the *PS* data to *PP* time, but that the variable γ tests are not as good as the constant γ of 0.6. Perhaps the actual V_P/V_S background is slowly varying and different from that calculated from the initial model.

	Zp	Zs	Density
Variable gamma. PP time	473	320	0.067
Variable gamma. PS time	457	317	0.065
gamma = 0.6. PP time	428	300	0.064
gamma = 0.6. PS time	419	286	0.063

Table 1. The errors between the original and predicted logs for each test.

Figure 6 shows the comparison between the real log curves (in blue) and the inverted results (in red) for the best case (*PS* time, $\gamma = 0.6$). The RMS error is displayed above the curve.

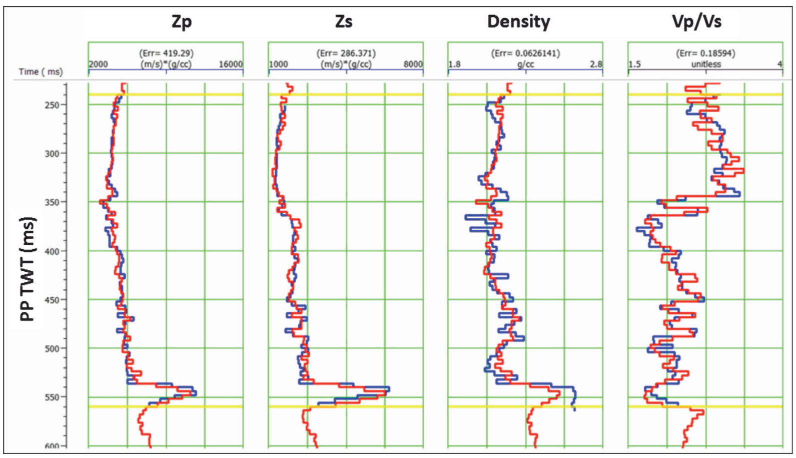


Figure 6. The comparison between the real log curves (in blue) and the inverted results (in red) for the best case (*PS* time, $\gamma = 0.6$).

Finally, Figure 7 shows the final inverted V_P/V_S ratio extracted from the seismic data using joint inversion between the *PP* and *PS* stacks.

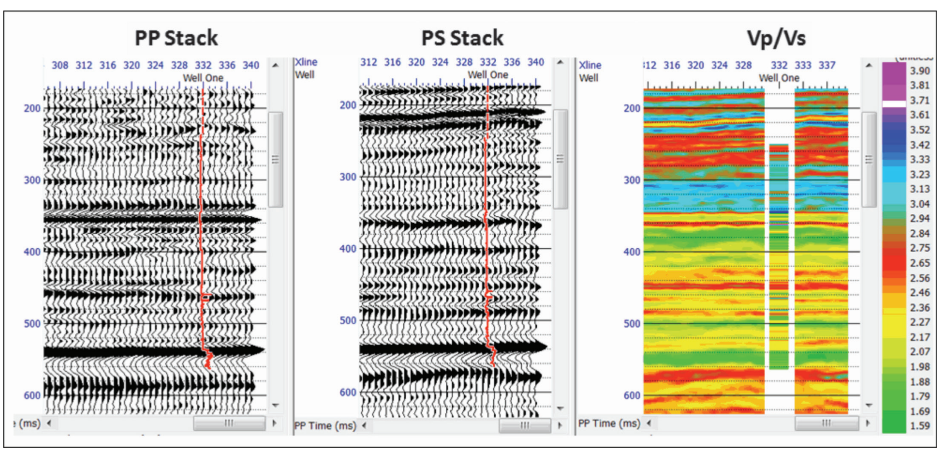


Figure 7. The *PP* stack, *PS* stack and final inverted V_P/V_S ratio extracted from the seismic data using joint inversion between the *PP* and *PS* stacks.

In the inversion result shown on the right of Figure 7, note that a reasonably good fit has been found between inserted well log values and the inverted seismic values. If pre-stack data had been available to us, we would have expected the fit to be even better, as seen from our synthetic example in the previous section.

CONCLUSIONS

We have presented a new approach to the joint simultaneous inversion of pre-stack seismic data which produces estimates of P-impedance, S-Impedance and density. This method allows us to incorporate both PP and PS data into the solution, if we have first calibrated both datasets to PP time. The method is based on three assumptions: that the linearized approximation for reflectivity holds, that reflectivity as a function of angle can be given by the Aki-Richards equations, and that there is a linear relationship between the logarithm of P-impedance and both S-impedance and density. Our approach was shown to work well for modelled gas and wet sands, for a real PP seismic example which consisted of a shallow Createaceous gas sand from Alberta, and for a joint PP-PS dataset from northeast Alberta. In the first two data examples, pre-stack PP seismic data was used. In the joint PP-PS dataset the prestack data was not available and we applied the method to full stacks. Future work will involve the incorporation of pre-stack PS data.

REFERENCES

Aki, K., and Richards, P.G., 2002, Quantitative Seismology, 2nd Edition: W. H. Freeman and Company.

Buland, A. and Omre, H, 2003, Bayesian linearized AVO inversion: Geophysics, 68, 185-198.

Margrave, G.F., Stewart, R. R. and Larsen, J. A., 2001, Joint PP and PS seismic inversion: The Leading Edge, 20, no. 9, 1048-1052.

Russell, B. and Hampson, D., 1991, A comparison of post-stack seismic inversion methods: Ann. Mtg. Abstracts, Society of Exploration Geophysicists, 876-878.

Simmons, J.L. and Backus, M.M., 1996, Waveform-based AVO inversion and AVO prediction-error: Geophysics, 61, 1575-1588.

ABOUT AUTHOR

Brian holds a B.Sc. from the University of Saskatchewan (1976), a M.Sc. from Durham University (1978), U.K., and a Ph.D. from the University of Calgary (2004), all in geophysics. He joined Chevron in Calgary as an exploration geophysicist in 1976 and subsequently worked for Teknica and Veritas before co-founding Hampson-Russell Software with Dan Hampson in 1987. Hampson-Russell is now a subsidiary of CGG, where Brian is Vice President, GeoSoftware and a CGG Fellow. Brian is involved in the research of new AVO, rock physics and inversion techniques as well as giving courses and talks throughout the world. He is a Past-President of both the SEG and Canadian SEG (CSEG) and has received Honorary Membership from both societies, as well as the Cecil Green Enterprise Award from SEG (jointly with Dan Hampson). Brian is also an Adjunct Professor in the Department of Geoscience at the University of Calgary and is registered as a Professional Geophysicist in the Province of Alberta.